Why gravitational waves cannot exist!

J-F Pommaret from CERMICS, Ecole des Ponts ParisTech, France, investigates the idea that gravitational waves cannot exist

1) A very long search:

In 1969, I decided to become a visiting student of D. C. Spencer at Princeton University and to apply these new tools in General Relativity (GR). A book published in 1978 from my PhD thesis and translated into Russian, started my research work. By analogy with Maxwell's equations for electromagnetism (EM), to deciding about the existence of a potential for Einstein equations in a vacuum has been proposed in the meantime as a $1,000 challenge by J. Wheeler, a friend of Spencer.

No progress was made during the next 25 years, until I gave a negative answer in 1995, contrary to what the GR community believed. Wheeler sent me back a letter with a one-dollar bill attached, refusing to admit this result. Indeed, while teaching elasticity, I proposed an exercise explaining why a dam made with concrete is *always* vertical on the water-side with a slope of about 42 degrees on the other free side in order to obtain a minimum cost and the auto-stability under cracking of the surface under water (See the Introduction of [2] and Zbl 1079.93001). The main tool was the approximate computation of the Airy function inside the dam. I discovered that the Airy parametrization was just the adjoint of the (linearized) *Riemann* operator used to generate compatibility condition (CC) for the deformation tensor by any engineer. Being involved in GR with A. Lichnerowicz, I got the idea of using the adjoint of an operator in a systematic way.

Then I found the recently published Master's thesis of the Japanese student M. Kashiwara. It has been a shock to discover this mixing up of differential geometry and homological algebra, culminating in the use of the *Differential Extension Modules.* In particular, if $D\xi = \eta$ has the generating CC $D_1\eta = 0$, then (D) may not generate all the CC of (D_1) and $ext^1(M)$ "*measures*" this gap only depending on the differential module M defined by D [2, 3]. Hence, exactly like homological algebra brought a *revolution* in mathematics, it will bring a *revolution* in physics. I also noticed that GR could be considered as "a" way to parametrize the *Cauchy = ad(Killing)* operator, leading to Gravitational Waves (GW).

It follows that the same confusion has been done by E. Beltrami (1892) and A. Einstein (1915) because they both used the same *Einstein* operator, *not knowing it was self-adjoint.*

Accordingly and until now, the GR community has never wanted to take these new tools into account and [6] provides a good example of such a poor situation both with the reason for which no other reference can be given. By chance, the control community has been interested during a while by these new techniques for studying OD or PD control systems with constant coefficients, thanks to U. Oberst. Hence, the impossibility to parametrize Einstein equations in a vacuum can only be found in books on control theory [Springer LNCIS 256, 2000 and 311, 2005].

Studying the Lanczos problems in 2001, I discovered that the *Beltrami = ad(Riemann)* operator can be parametrized by the *Lanczos = ad(Bianchi)* operator in the adjoint sequence. As a byproduct, the purpose of this pamphlet is to explain *without any computation,* the previous confusion between the *Cauchy = ad(Killing)* operator and the *Bianchi* operator. According to H. Poincaré, the geometrical and physical long exact dual differential sequences of operators acting on tensors, giving order of operators and number of components, are:

$$\begin{array}{ccccccc}
n & \underset{1}{\overset{Killing}{\longrightarrow}} & \frac{n(n+1)}{2} & \underset{2}{\overset{Riemann}{\longrightarrow}} & \frac{n^2(n^2-1)}{12} & \underset{1}{\overset{Bianchi}{\longrightarrow}} & \frac{n^2(n^2-1)(n-2)}{24} \\
n & \underset{1}{\overset{Cauchy}{\longleftarrow}} & \frac{n(n+1)}{2} & \underset{2}{\overset{Beltrami}{\longleftarrow}} & \frac{n^2(n^2-1)}{12} & \underset{1}{\overset{Lanczos}{\longleftarrow}} & \frac{n^2(n^2-1)(n-2)}{24}
\end{array}$$

2) A basic control example:

Let a rigid bar be able to move horizontally with reference position x and attach two pendulums with lengths l_1 and l_2 making the (small) angles θ1 and θ2 with the vertical. The system for $\eta = (x, \theta^1, \theta^2)$ with gravity g is $D_1\eta = 0$:

$$d^2x + l_1 d^2\theta^1 + g\theta^1 = 0, \ d^2x + l_2 d^2\theta^2 + g\theta^2 = 0$$

With a little skill, one can stop *any* movement by just moving the bar horizontally along itself iff l_1 ≠/ l_2. Equivalently, the system is controllable iff the operator (D_1) is injective, without using the Kalman test (1960) [3].

Multiplying the equations by λ^1 and λ^2, adding and integrating by parts, one gets (D_1)λ = 0, namely:

$$d^2\lambda^1 + d^2\lambda^2 = 0, \ l_1 d^2\lambda^1 + g\lambda^1 = 0, \ l_2 d^2\lambda^2 + g\lambda^2 = 0$$

Differentiating twice, one may find iff l_1 ≠/ l_2:

$$l_2\lambda^1 + l_1\lambda^2 = 0, \ (l_2/l_1)\lambda^1 + (l_1/l_2)\lambda^2 = 0 \Rightarrow \lambda = 0$$

One finally obtains a fourth order (!) parametrization $D\varphi = \eta$:

$$-l_1l_2d^4\phi - g(l_1+l_2)d^2\phi - g^2\phi = x,\ l_2d^4\phi + gd^2\phi = \theta^1,\ l_1d^4\phi + gd^2\phi = \theta^2$$

This parametrization is injective and we have the *short* exact *dual* sequences:

$$\begin{array}{ccccccccc} 0 & \longrightarrow & 1 & \underset{4}{\overset{\mathcal{D}}{\longrightarrow}} & 3 & \underset{2}{\overset{\mathcal{D}_1}{\longrightarrow}} & 2 & \longrightarrow & 0 \\ 0 & \longleftarrow & 1 & \underset{4}{\overset{ad(\mathcal{D})}{\longleftarrow}} & 3 & \underset{2}{\overset{ad(\mathcal{D}_1)}{\longleftarrow}} & 2 & \longleftarrow & 0 \end{array}$$

If $l_1 = l_2 = l$, then $z = \theta^1 - \theta^2$ is satisfying the OD equation $ld^2z + gz = 0$. Contrary to what most engineers believe, I proved in 1995 [2, 3] that CONTROLLABILITY IS A STRUCTURAL PROPERTY OF A CONTROL SYSTEM THAT DOES NOT DEPEND ON THE CHOICE OF THE INPUTS AND OUTPUTS AMONG THE SYSTEM VARIABLES.

3) Differential double duality:

The following constructive test with 5 steps largely supersedes the Kalman test [2, 3, 7, 8]: Start with D_1, construct (D_1), then find its CC in the form of an operator (D). Finally, denoting by D_1' the CC of = D, the parametrization exists if, and only if we may have $D_1' = D_1$. Indeed, as ° $D = 0$, then D_1 is surely among the CC of D but other CC may also exist along the following diagram:

$$\begin{array}{ccccccc} & & & & & \zeta' & \boxed{5} \\ & & & & \overset{\mathcal{D}_1{}'}{\nearrow} & & \\ \boxed{4} & \xi & \overset{\mathcal{D}}{\longrightarrow} & \eta & \overset{\mathcal{D}_1}{\longrightarrow} & \zeta & \boxed{1} \\ \boxed{3} & \nu & \overset{ad(\mathcal{D})}{\longleftarrow} & \mu & \overset{ad(\mathcal{D}_1)}{\longleftarrow} & \lambda & \boxed{2} \end{array}$$

One can prove that each new CC brought by D'_1 that is not already a differential consequence of D_1 provides a quantity satisfying at least one OD or PD equation for *itself*.

4) Beltrami (1892) versus Einstein (1915):

Linearizing the *Ricci* tensor *ρij* over the Minkowski metric ω, we obtain the *Ricci* operator for the perturbation Ω of ω:

$$2R_{ij} = \omega^{rs}(d_{rs}\Omega_{ij} + d_{ij}\Omega_{rs} - d_{ri}\Omega_{sj} - d_{sj}\Omega_{ri}) = 2R_{ji} \Rightarrow tr(R) = \omega^{ij}R_{ij}$$

with 4 terms and the *Einstein* operator by setting *Eij* = *Rij* − (1/2)*ωijtr(R*) with 6 terms.

When $n \geq 3$, the right part of the Killing resolution of the first section projects onto:

$$\frac{n(n+1)}{2} \xrightarrow[2]{Einstein} \frac{n(n+1)}{2} \xrightarrow[1]{div} n \rightarrow 0$$

The *Einstein* operator is self-adjoint (a crucial property for which I don't know any reference !!!), and we may get successively the five steps with *ad(Einstein)* = *Einstein*:

$$\boxed{1}\, \mathcal{D}_1 = Einstein, \boxed{2}\, Einstein, \boxed{3}\, ad(Killing) = Cauchy, \boxed{4}\, \mathcal{D} = Killing, \boxed{5}\, \mathcal{D}'_1 = Riemann$$

We obtain the strict symbolic inclusion $D_1 \subset D'_1$ in the diagram existing when $n = 4$:

$$\begin{array}{ccccccccccc}
 & & & & & & 20 & \xrightarrow{Bianchi} & 20 & & \\
 & & & & & \nearrow^{Riemann} & \downarrow & & \downarrow & & \\
 & & 4 & \xrightarrow{Killing} & 10 & \xrightarrow{Einstein} & 10 & \xrightarrow{div} & 4 & \longrightarrow & 0 \\
0 & \longrightarrow & 4 & \xleftarrow{Cauchy} & 10 & \xleftarrow{Einstein} & 10 & & & &
\end{array}$$

The Cauchy and *Killing* operators (*left side*) have thus *strictly nothing* to do with the *Bianchi* and therefore *div* operators *(right side)*. In addition, the 10 stress potentials are no longer tensors but tensor densities and have nothing to do with the perturbation Ω of the metric. According to section 3, the 20 − 10 = 10 new CC are generated by the 10 independent components of the Weyl tensor, each one being killed by the D'Alembertian, a striking result *totally unknown* in this framework!!!

Already in 2017, I proved that GW cannot exist, not because of a problem of DETECTION but because their EQUATION is just the *ad(Ricci)* operator with the same previous comments.

When $n = 2$ in plane elasticity, one has (Compare to the double pendulum!):

$$\begin{array}{ccccccccc}
 & & 2 & \xrightarrow[1]{Killing} & 3 & \xrightarrow[2]{Riemann} & 1 & \longrightarrow & 0 \\
0 & \longleftarrow & 2 & \xleftarrow[1]{Cauchy} & 3 & \xleftarrow[2]{Airy} & 1 & &
\end{array}$$

Multiplying the *Riemann* operator $D_1 : \Omega \rightarrow d_{22}\, \Omega 11 + d_{11}\, \Omega 22 - 2d12\, \Omega 12$ by a test function φ *and integrating by parts, we obtain the Airy = ad(Riemann)* (Who knows !) parametrization $\sigma 11 = d_{22}\, \varphi$, $\sigma^{12} = \sigma^{21} = -d12\, \varphi$, $\sigma^{22} + d_{11}\varphi$ of the *Cauchy* operator provided in 1863.

When $n = 3$, E. Beltrami introduced in 1892 the 6 stress functions $\varphi ij = \varphi ji$ in the self-adjoint *Beltrami = ad(Riemann) parametrization*. The identification *Lanczos = ad(Bianchi)* leads to the long exact dual sequences, with the same confusion as Einstein but … 25 years before:

$$
\begin{array}{ccccccccccc}
 & & 3 & \underset{1}{\overset{Killing}{\longrightarrow}} & 6 & \underset{2}{\overset{Riemann}{\longrightarrow}} & 6 & \underset{1}{\overset{Bianchi}{\longrightarrow}} & 3 & \longrightarrow & 0 \\
0 & \longleftarrow & 3 & \underset{1}{\overset{Cauchy}{\longleftarrow}} & 6 & \underset{2}{\overset{Beltrami}{\longleftarrow}} & 6 & \underset{1}{\overset{Lanczos}{\longleftarrow}} & 3 & &
\end{array}
$$

5) General relativity and gauge theory: Beyond the mirror!

Only the bottom row and the right column are known in the following commutative and exact algebraic Fundamental diagram II of tensors that I found in 1983 (1, p 446). A diagonal *snake chase* proves that *Ricci* ≃ S_2T^* wheb $\hat{g}_2$ is the second order symbol of the infinitesimal Lie equations of the conformal group of space-time (8,9). This result explains the confusions done by A. Einstein and H. Weyl in their tentatices to use the lower sequence for linking GR and EM, through the splitting $T^* \otimes T^* \simeq S_2T^* \oplus \wedge^2 T^* \simeq (Rij) \oplus (Fij)$ and the Spencer δ-cohomology:

$$
\begin{array}{ccccccccccc}
 & & & & & & & & 0 & & \\
 & & & & & & & & \downarrow & & \\
 & & & & & & 0 & & Ricci & & \\
 & & & & & & \downarrow & & \downarrow & & \\
 & & & & 0 & \longrightarrow & Z_1^2(g_1) & \longrightarrow & Riemann & \longrightarrow & 0 \\
 & & & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & \longrightarrow & T^* \otimes \hat{g}_2 & \overset{\delta}{\longrightarrow} & Z_1^2(\hat{g}_1) & \longrightarrow & Weyl & \longrightarrow & 0 \\
 & & & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & S_2T^* & \overset{\delta}{\longrightarrow} & T^* \otimes T^* & \overset{\delta}{\longrightarrow} & \wedge^2 T^* & \longrightarrow & 0 & & \\
 & & & & \downarrow & & \downarrow & & & & \\
 & & & & 0 & & 0 & & & &
\end{array}
$$

Such a *mathematical fact* is in *total contradiction* with the use of the unitary group $U(1)$ in Gauge Theory (GT) which is not acting on space-time. Paraphrasing W. Shakespeare, we may finally say:

"TO ACT OR NOT TO ACT, THAT IS THE QUESTION!"

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